Shortest Paths in Networks with Correlated Link Weights

Song Yang, Stojan Trajanovski and Fernando A. Kuipers
Delft University of Technology, The Netherlands

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Introduction

• In many real-life networks the link weights (e.g., delay, failure probability) are correlated:
  • Multi-layer or overlay networks
  • Inter-dependent networks
  • Shared-risk link group (SRLG) networks
A Multilayer Network Example

- Links (s,a) and (a,t) in the logical layer are correlated since they both traverse link (s,b) in the physical layer.

- The shortest path from s to t is 5, which is not equal to 8+6=14.
The Impact of Link Correlation on Path Calculation.

Links (s,a) and (b,t) are correlated with joint cost 11.

Cost of path s-a-b-t: 11+4=15 which is not equal to 6+10+4=20.

Cost of path s-b-t: 8+10=18
Correlated Link Weight Models

• Deterministic Correlated Model
  \[ \omega(l_1) \oplus \omega(l_2) = \rho_{12} \cdot (\omega(l_1) + \omega(l_2)) \quad \oplus: \text{joint cost operator} \]
  \( \rho_{12} \) is called as the correlated coefficient, and ranges in \((0,\infty)\)
  .
  \( 0 < \rho < 1: \text{decreasing correlation}, \quad \rho > 1: \text{increasing correlation}, \quad \rho = 1: \text{non-correlated} \)

  \[ \omega(l_1) \oplus \omega(l_2) \oplus \ldots \oplus \omega(l_k) = \rho \cdot (\omega(l_1) + \omega(l_2) + \ldots + \omega(l_k)) \]

• Nodal Deterministic Correlated Model
  .
  Only the links share one common node follow the deterministic correlated model
Stochastic link weight

- In many real-life networks, the link weights are uncertain because of inaccurate Network State Information (NSI), network dynamics, failure and maintenance events, etc.

- Many common distributions (e.g., Exponential distribution, Uniform distribution, etc.) are log-concave:
  \[ \log f(\theta x + (1-\theta)y) \geq \theta \log f(x) + (1-\theta)\log f(y) \]

Assume the link cost follows a log-concave distribution. The probability of allocating at most \(d\) cost is \(CDF(d)\).
Stochastic Correlated Model

• The joint Cumulative Density Function (CDF) is given and assumed to be log-concave (e.g., multivariate Normal Distribution)

•  \( CDF(x_1, x_2) \): the joint probability for links \( l_1 \) and \( l_2 \) to allocate \( x_1 \) and \( x_2 \) costs, respectively.

•  \( CDF(x_1, x_2, \ldots, x_k) \): the joint probability for links \( l_1, l_2, \ldots, l_k \) to allocate at most \( x_1, x_2, \ldots, x_k \) costs, respectively.
A two-dimensional multivariate Normal Distribution Example

For both variables:
Mean: 2
Range: [0, 4]

Covariance matrix:

\[
\begin{bmatrix}
0.9 & 0.4 \\
0.4 & 0.3 \\
\end{bmatrix}
\]
Shortest Path under the Deterministic Correlated Model (SPDCM) Problem

- Given
  - a directed network $G(N,L)$
  - each link $l$ has a cost $w(l)$ following the deterministic correlated model.

- Problem
  - find a path from the source $s$ to the destination $t$ with minimum cost.
Complexity and solution of the SPDCM Problem and its variant

- The SPDCM problem is NP-hard.
- The SPDCM problem cannot be approximated to arbitrary degree, unless P=NP.
- We devise a brute-force algorithm to solve the SPDCM problem exactly.
- A conventional shortest path can have cost at most
  $\max\left(\frac{\rho_{\text{max}}}{\rho_{\text{opt}}}, \frac{1}{\rho_{\text{min}}}\right) \cdot \text{opt}$
- The Shortest Path under the Nodal Deterministic Correlated Model (SPNDCM) Problem is polynomial-time solvable
Graph transformation to solve the SPNDCM problem

Original graph

Auxiliary graph
Shortest Path under the Stochastic Correlated Model (SPSCM) Problem

- **Given**
  - A directed graph $G(N,L)$
  - the link costs follow the stochastic correlated model

- **Problem**
  - find a path from the source $s$ to the destination $t$

- **Constraints**
  - its total cost is minimized and
  - the probability to realize this value is no less than $P_s$
Shortest Path under the Stochastic Correlated Model (SPSCM) Problem

• We propose a convex optimization formulation to solve the SPSCM problem

• A convex optimization problem is a problem in which the objective function is either a maximization of a concave function or a minimization of a convex function, and its constraints are all convex.

• Convex optimization problems can usually be solved quickly and accurately with convex optimization solvers.

• Most of the convex optimization problems are polynomial time solvable.
Conclusion

- We have proposed two correlated link weight models, namely, (1) deterministic correlated model and (2) stochastic correlated model.

- We have proved that, the shortest path under the deterministic correlated model is NP-hard, and cannot be approximated in polynomial time, unless P=NP.

- We have shown that the shortest path under the nodal deterministic correlated model is polynomial-time solvable.

- We have propose a convex optimization formulation to solve the shortest path under the stochastic correlated model.

- An extension of the Min-Cut problem in the proposed models is also studied in our arXiv paper: *Optimization Problems in Correlated Networks*, arXiv: 1502.06820.
Questions?